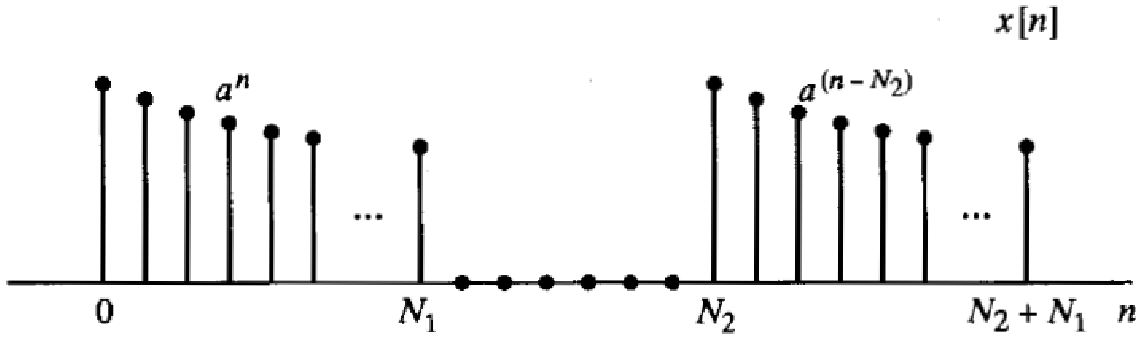
ASSIGNMENT 2

1. A linear time invariant system has impulse response h[n] = u[n]. Determine the response of this system input x[n] shown in the figure below and described as

$$x[n] = \begin{cases} 0, & n < 0 \\ a^n, & 0 \le n \le N_1 \\ 0, & N_1 < n < N_2 \\ a^{n-N_2}, & N_2 \le n \le N_2 + N_1 \\ 0, & N_2 + N_1 < n \end{cases}$$

Where $0 < a < 1$.



2. Which of the following discrete time functions could be eigenfunctions of any stable LTI System?

(a) $5^n u[n]$

(b) $e^{j2\omega n}$

(c) $e^{j\omega n} + e^{j2\omega n}$

(d) 5^n

(e) $5^n . e^{j2\omega n}$

3. For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, and (4) time invariant.

(a) $T(x[n]) = (cos\pi n)x[n]$

(b) $T(x[n]) = x[n^2]$

(c) $T(x[n]) = x[n]\sum_{k=0}^{\infty} \delta[n-k]$

(d) $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$

4. Consider the difference equation

$$y[n] + \frac{1}{15}y[n-1] - \frac{2}{5}y[n-2] = x[n]$$

(a) Determine the general form of the homogeneous solution to this equation.

(b) Both a causal and an anti-causal LTI system are characterized by the given difference equation. Find the impulse responses of the two systems.

(c) Show that the causal LTI system is stable and the anti-causal LTI system is unstable.

(d) Find a particular solution to the difference equation when $x[n] = \left({}^{3}/_{5}\right)^{n} u[n]$.

5. A linear time-invariant system has impulse response $h[n] = a^n u[n]$.

(a) Determine $y_1[n]$, the response of the system to the input $x_1[n]=e^{j\left(\frac{\pi}{2}\right)n}$

(b) Use the result of part(a) to help to determine $y_2[n]$, the response of the system to the input $x_2[n]=\cos(\pi n/2)$.

(c) Determine $y_3[n]$, the response of the system to the input $x_3[n] = e^{j\left(\frac{\pi}{2}\right)n}\, u[n]$.

(d) Compare $y_3[n]$ with $y_1[n]$ for large n .

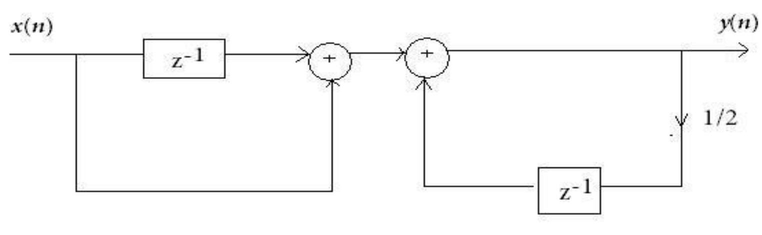
6. Consider the system shown in Figure

(a) Determine it's impulse response $h(n)$.

(b) Show that $h(n)$ is equal to the convolution of following two signals.

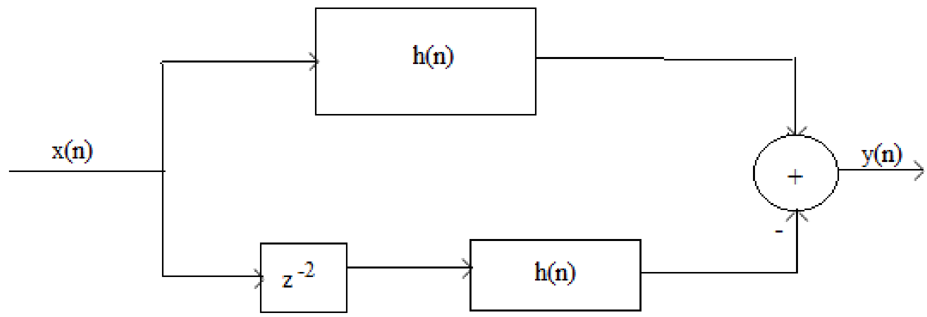
$$h_1(n) = \delta(n) + \delta(n-1)$$

$$h_2(n) = ((1/2)^n)\, u[n].$$



7 Consider the system in figure with $h(n) = a^n u(n)$, $-1<a<1$. Determine the response $y(n)$ of the system to the excitation

$$x(n) = u(n+5) - u(n-10)$$



8 Compute the zero-state response of the system described by the difference equation

$$y(n) + ½\, y(n-1) = x(n) + 2x(n-2)$$

to the input

$$x(n) = \{1,2,3,4,2,1\}$$

↑

by solving the difference equation recursively.

9. Two discrete- time systems T_1 and T_2 are connected in cascade to form a new system T as shown in fig. Prove or disprove the following statements.

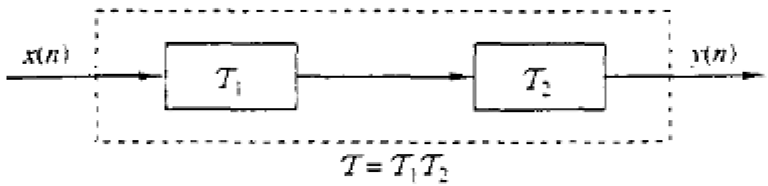
a) If T_1 and T_2 are linear, then T is linear (i.e. the cascade connection of two linear system is linear).
b) If T_1 and T_2 are time invariant, then T is time invariant.
c) If T_1 and T_2 are causal, then T is causal.
d) If T_1 and T_2 are linear and time invariant, the same holds for T.
e) If T_1 and T_2 are linear and time variant, then interchanging their order does not change the system.

f) As in part (e) except that T_1 and T_2 are now time varying. (Hint: Use an example.)

g) If T_1 and T_2 are nonlinear, then T is nonlinear.

h) If T_1 and T_2 are stable, then T is stable.

i) Show by an example that the inverse of parts (c) and (h) do not hold in general.



10. Consider a system with impulse response

$$h(n) = \begin{cases} (1/2)^n, & n \le 4 \\ 0, & otherwise \end{cases}$$

Determine the input *x(n)* for 0≤n≤8 that will generate the output sequence

y(n)= {1 2 2 5 3 3 3 2 1 0 ……..}

↑

MATLAB PROBLEM

1.When the sequences *x(n)* and *h(n)* are of finite duration N_x and N_h , respectively, then their linear convolution

$$y(n) = LTI\,[x(n)] = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

can also be implemented using *matrix-vector multiplication*. If the elements of *y(n)* and *x(n)* are arranged in the column vectors **x** and **y** respectively, then from the above equationwe obtain

y = Hx

where linear shifts in *h(n-k)* for $n = 0,\ldots, N_h$-1 are arranged as rows in the matrix **H**. This matrix has an interesting structure and is called a *Toeplitz matrix*. To investigate this matrix, consider the sequences

$$x(n) = \{1,2,3,4\} \quad \text{and} \quad h(n) = \{3,2,1\}$$

a. Determine the linear convolution *y(n) = h(n) * x(n)*
b. Express *x(n)* as a 4 x 1 column vector **x** and *y(n)* as a 6 x 1 column vector **y**. Now determine the 6 x 4 matrix **H** so that **y = Hx.**
c. Characterize the matrix **H**. From this characterization can you give a definition of a Toeplitz matrix? How does this definition compare with that of the time invariance?
d. What can you say about the first row and first column of **H**?

2. MATLAB provides a function called **toeplitz** to generate a Toeplitz matrix, given the first row and first column.

a. Using this function and your answer to 2.13 part(d), develop an alternate MATLAB function to implement linear convolution. The format of the function should be

```
function [y , H]  = conv_tp(h , x)
% Linear convolution using Toeplitz matrix
% --------------------------------------------------
% [y , H] = conv_tp(h , x)
%  y = output sequence in column vector form
% H = Toeplitz matrix corresponding to sequence h so that y = Hx.
%  h =  impulse response sequence in column vector form
%  x =  input sequence in column vector form
```

b. Verify your function on the sequences given in the problem 2.13.

3. The operation of *signal dilation* (or *decimation* or *down-sampling*) is defined by

$$y(n) = x(nM)$$

In which the signal $x(n)$ is down-sampled by an integer factor M. For example, if

$$x(n) = \{\dots, -2, 4, \underset{\uparrow}{3}, -6, 5, -1, 8, \dots\}$$

then the down-sampled sequences by a factor 2 are given by

$$y(n) = \{\dots, -2, \underset{\uparrow}{3}, 5, 8, \dots\}$$

a. Develop a MATLAB function **dnsample** that has the form

function y = dnsample(x,M)

to implement the above operation. Use the indexing mechanism of MATLAB with careful attention to the origin of the time axis *n*=0.

b. Generate $x(n) = sin(0.125\pi n), \quad -50 \leq n \leq 50$. Decimate $x(n)$ by a factor of 4 to generate $y(n)$. Plot both $x(n)$ and $y(n)$ using subplot and comment on the results.

c. Repeat the above using $x(n) = sin(0.5\pi n), \quad -50 \leq n \leq 50$. Qualitatively discuss the effect of down-sampling on signals.

4.The complex exponential sequence $e^{j\omega_0 n}$ or the sinusoidal sequence $\cos(\omega_0 n)$ are periodic if the *normalised* frequency $f_0 \triangleq \frac{\omega_0}{2\pi}$ is a rational number; that is, $f_0 = \frac{K}{N}$, where K and N are integers.

a. Prove the above result.

b. Generate and plot $\cos(0.3\pi n) - 20 \leq n \leq 20$. Is this sequence periodic? If it is, what is the fundamental period? From the examination of the plot, what interpretation can you give to the intergers K and N above?

c. Generate and plot $\cos(0.3n) - 20 \leq n \leq 20$. Is this sequence periodic? What do you conclude from the plot? If necessary, examine the values of the sequence in MATLAB to arrive at your answer.

5 Let $x(n) = (0.8)^n u(n)$

a. Determine $x(n) * x(n)$ analytically.

b. Using the filter function, determine the first 50 samples of $x(n) * x(n)$. Compare your results with those of part a.