EE301-Assignment4

1. The inverse z-transform of $X(z)$ is $x(n) = \left(\frac{1}{2}\right)^n u(n)$. Using the z-transform properties, determine the sequences in each of the following cases.

a. $X_1(z) = \frac{z-1}{z} X(z)$

b. $X_2(z) = z\,X(z^{-1})$

c. $X_3(z) = 2\,X(3z) + 3X\left(\frac{z}{3}\right)$

d. $X_4(z) = X(z)X(z^{-1})$

e. $X_5(z) = z^2 \frac{dX(z)}{dz}$

2. If sequences $x_1(n), x_2(n)$ and $x_3(n)$ are related by $x_3(n) = x_1(n) * x_2(n)$, then

$$\sum_{n=-\infty}^{\infty} x_3(n) = \left(\sum_{n=-\infty}^{\infty} x_1(n)\right)\left(\sum_{n=-\infty}^{\infty} x_2(n)\right)$$

a. Prove the above result by substituting the definition of convolution in the left-hand side.

b. Prove the above result by using the convolution property.

c. Verify the above result using MATLAB by choosing any two sequences $x_1(n)$ and $x_2(n)$.

3. For the linear and time-invariant systems described by the impulse responses below, determine (i) the system function representation, (ii) the difference equation representation, (iii) the pole-zero plots, and (iv) the output $y(n)$ if the input is $x(n) = \left(\frac{1}{4}\right)^n u(n)$.

a. $h(n) = 2\left(\frac{1}{2}\right)^n u(n)$

b. $h(n) = n\left(\frac{1}{3}\right)^n u(n) + \left(-\frac{1}{4}\right)^n u(n)$

c. $h(n) = 3(0.9)^n \cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right) u(n+1)$

d. $h(n) = n[u(n) - u(n-10)]$

e. $h(n) = [2 - sin(\pi n)]u(n)$

4. Determine the inverse Z-transform of each of the following. In Parts (a)-(c), use the methods specified. In Part (d), use any method you prefer.

(a) Long division:

$$X(z) = \frac{1-\frac{1}{3}z^{-1}}{1+\frac{1}{3}z^{-1}}, \qquad x[n] \text{ a right handed sequence}$$

(b) Partial fraction:

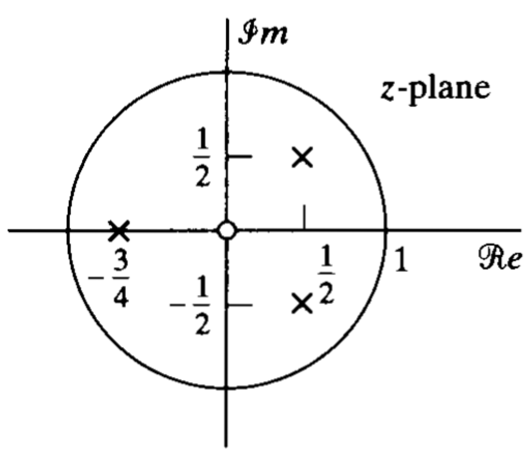
$$X(z) = \frac{3}{z-\frac{1}{4}-\frac{1}{8}z^{-1}}, \qquad x[n] \text{ stable}$$

(c) Power Series:

$$X(z) = \ln(1-4z), \quad |z| < \frac{1}{4}$$

(d) $X(z) = \frac{1}{1-\frac{1}{3}z^{-3}}, \qquad |z| > (3)^{-1/3}$

5. The pole-zero diagram in figure below corresponds to the z-transform *X(z)* of a casual sequence *x[n].* Sketch the pole-zero diagram of *Y(z)*, where *y[n] = x[-n+3].* Also, specify the region of convergence for *Y(z)*.



6. The deconv function is useful in dividing two causal sequences. Write a MATLAB function deconv.m to divide two noncausal sequences (similar to the conv function). The format of this function should be

```
function [p,np,r,nr] = deconv_m(b,nb,a,na)
%      Modified deconvolution routine for noncausal sequences
%      function [p,np,r,nr] = deconv_m(b,nb,a,na)%
%      p = polynomial part of support np1<= n<= np2
%      np = [np1, np2]
%      r = remainder part of support nr1<= n<= nr2
%      nr= [nr1, nr2]
%      b = numerator polynomial of support nb1<= n<= nb2
%      nb = [nb1, nb2]
%      a = denominator polynomial of support na1<= n<= na2
%      na = [na1, na2]
```

Check your function on the following operation

$$\frac{z^2+z+1+z^{-1}+z^{-2}+z^{-3}}{z+2+z^{-1}} = (z+1+2z^{-1}-2z^{-2}) + \frac{3z^{-2}+3z^{-3}}{z+2+z^{-1}}$$

7. Suppose *X(z)* is given as follows:

$$X(z) = \frac{2+3z^{-1}}{1-z^{-1}+0.81z^{-2}}, \qquad |z| > 0.9$$

a. Determine *x(n)* in a form that contains no complex numbers

b. Using MATLAB, find the first 20 samples of *x(n)* and compare them with your answer in the above part.

8. A stable system has the following pole-zero locations:

$$z_{1}=j, \qquad z_{2}=-j, \qquad p_1 = -\frac{1}{2}+j\frac{1}{2}, \qquad p_2 = -\frac{1}{2}-j\frac{1}{2}$$

It is also known that the frequency response function $H(e^{j\omega})$ evaluated at ω=0 is equal to 0.8, that is $H\left(e^{j0}\right) = 0.8$

a. Determine the system function H(z) and indicate its region of convergence.

b. Determine the difference equation representation.

c. Determine the steady-state response $y_{ss}(n)$ if the input is $x(n) = \frac{1}{\sqrt{2}} \sin\left(\frac{\pi n}{2}\right) u(n)$

d. Determine the transient response $y_{tr}(n)$ if the input is $x(n) = \frac{1}{\sqrt{2}} \sin\left(\frac{\pi n}{2}\right) u(n)$