EE301-Assignment5

1. A 12-point sequence is $x(n)$ defined as

$$x(n) = \{ 1,2,3,4,5,6,6,5,4,3,2,1 \}$$

a. Determine the DFT $X(k)$ of $x(n)$. Plot (using the stem function) its magnitude and phase.

b. Plot the magnitude and phase of the DTFT $X(e^{jw})$ of $x(n)$ using MATLAB.

c. Verify that the above DFT is sampled version of $X(e^{jw})$ **.** It might be helpful to combine the above two plots in one graph using the hold function.

d. Is it possible to reconstruct the DTFT $X(e^{jw})$ from the DFT $X(k)$? If possible, give the necessary interpolation formula for reconstruction. If not possible, state why this reconstruction cannot be done.

2. Let $H(e^{jw})$ be the frequency response of a real, causal discrete-time LTI system.

a. If Re$\{H(e^{jw})\}$ = $\sum_{k=0}^{5}(0.5)^k\cos(kw)$ determine the impulse response h(n) analytically. Verify your answer using IDFT as a computational tool. Choose the length N judiciously.

b. If Im$\{H(e^{jw})\} = 2l\ \sin(lw)$ and $\int_{-\pi}^{\pi} H(e^{jw})dw$ = 0 determine the impulse response h(n) analytically. Verify your answer using IDFT as a computational tool. Choose the length N judiciously.

3. Let $X(k)$ denote the *N*-point DFT of an *N*-point sequence $x(n)$. The DFT $X(k)$ itself is an *N*-point sequence.

a. If the DFT of $X(k)$ is computed to obtain another *N*-point sequence $x_1(n)$, show that

$$x_1(n) = Nx((n))_N, \quad 0 \le n \le N-1$$

b. Using the above property, design a MATLAB function to implement an *N*-point circular folding operation $x_2(n) = x_1((-n))_N$. The format should be x2 = circfold (x1, N)

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% Circular folding using DFT
% x2 = circfold (x1,N)
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% x2 = circulatory folded output sequence
% x1 = input sequence of length <= N
% N = circular buffer length

c. Determine the circular folding of the following sequence:

$x_1(n)$ = { 1,2,3,4,5,6,6,5,4,3,2,1}

4. Using the frequency domain approach, develop a MATLAB function to determine a circular shift $x((n-m))_N$, given an N_1 – point sequence $x(n)$, where $N_1 \leq N$. Your function should have following format.

Function $y = cirshftf(x, m, N)$

% Function $y = cirshftf(x, m, N)$
%
% Circular shift of m samples wrt size N in sequence x: (Frequency domain)
%---
% y: output sequence containing the circular shift
% x: input sequence of length ≤ N
% m: sample shift
% N: size of circular buffer
% Method: $y(n) = idft(dft(x(n)) * WN\text{^}(mk))$
%
% If m is scalar then y is a sequence (row vector)
% If m is vector then y is a matrix , each row is circular shift in x corresponding to entries
% in vector m
% m and x should not be matrices

Verify your function on the following sequence $x_1(n) = 11 - n\,,\; 0 \leq n \leq 10$ with $m = 10$ and $= 15$.

5. Using the frequency domain approach, develop a MATLAB function to implement circular convolution operation between two sequences. The format of the function should be

Function $x3 = circonvf(x1, x2, N)$

% Circular convolution in the frequency domain
% $x3 = circonvf(x1, x2, N)$
% $x3$ = convolution result of length N
% $x1$ = sequence of length ≤ N
% $x2$ = sequence of length ≤ N
% N = length of circular buffer

6. Given the sequences $x_1(n)$ and $x_2(n)$ shown below
$x_1(n) = \{2,1,1,2\}$, $x_2(n) = \{1,-1,-1,1\}$

a. Compute a circular convolution $x_1(n) \boxed{N} x_2(n)$ for N = 4,7 and 8.

b. Compute linear convolution $x_1(n) * x_2(n)$.

c. Using results of calculations, determine minimum value of N so that linear and circular convolutions are the same on the N – point interval.

d. Without performing the actual convolutions, explain how you could have obtained the result of part c.

7. Using the equation,

$$X(p,q) = \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} x(l,m) W_N^{(Ml+m)(p+Lq)}$$

Determine and draw the signal flow graph for the N=8 point, radix-2 decimation-in-frequency FFT algorithm. Using this flow graph, determine the DFT of the sequence

$$x(n) = \cos\left(\frac{\pi n}{2}\right), \qquad 0 \le n \le 7$$

8. Using the equation,

$$X(p,q) = \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} x(l,m) W_N^{(Ml+m)(p+Lq)}$$

Determine and draw the signal flow graph for the N=16 point, radix-4 decimation-in-time FFT algorithm. Using this flow graph, determine the DFT of the sequence

$$x(n) = \cos\left(\frac{\pi n}{2}\right), \qquad 0 \le n \le 15$$

9. Let *x(n) = cos (πn/99), 0 ≤ n ≤ (N-1)* be an N-point sequence. Choose N = 4^γ and determine the execution times in MATLAB for γ = 5, 6, … … ., 10. Verify that these times are proportional to $N\log_4 N$