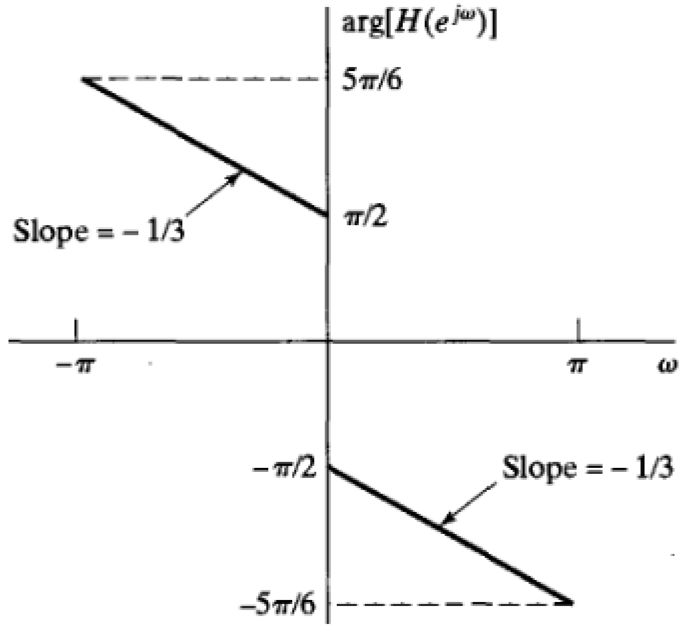
Assignment 3

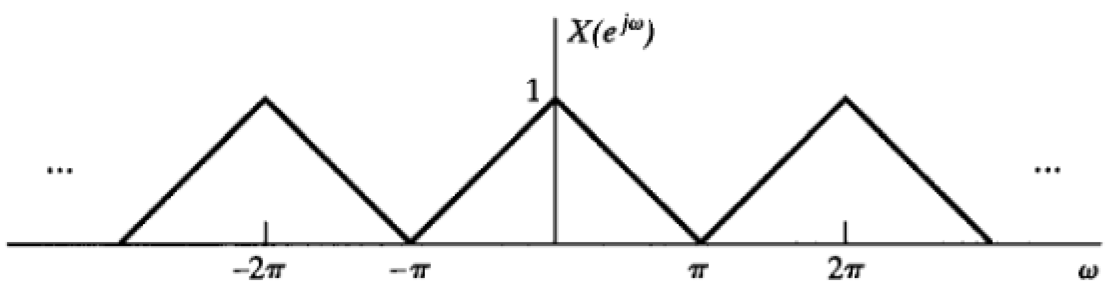
1. Consider an LTI system with $|H(e^{jw})|=1$ and let $\arg[H(e^{jw})]$ be as shown in fig if the input is

$$x[n] = \cos(3\pi n/2 + \pi/4)$$

determine the output $y[n]$



2. Let $x[n]$ and $X(e^{j\omega})$ represent a sequence and its Fourier transform, respectively. Determine, in terms of $X(e^{j\omega})$, the transforms of $y_s[n]$, $y_d[n]$, and $y_e[n]$. In each case, sketch $Y(e^{j\omega})$ for $X(e^{j\omega})$ as shown in Figure.



(a) Sampler:

$$y_s[n] = \begin{cases} x[n], & n \text{ even}, \\ 0, & n \text{ odd}. \end{cases}$$

Note that $y_s[n] = \frac{1}{2}\{x[n] + (-1)^n x[n]\}$ and $-1 = e^{j\pi}$.

(b) Compressor:

$$y_d[n] = x[2n].$$

(c) Expander:

$$y_e[n] = \begin{cases} x[n/2], & n \text{ even}, \\ 0, & n \text{ odd}. \end{cases}$$

3. A sequence has the discrete-time Fourier transform

$$X(e^{j\omega}) = \frac{1 - a^2}{(1 - ae^{-j\omega})(1 - ae^{j\omega})}, \quad |a| < 1.$$

(a) Find the sequence $x[n]$.

(b) Calculate $\int_{-\pi}^{\pi} X(e^{j\omega}) \cos(\omega)\, d\omega/2\pi$.

4. The LTI system

$$H(e^{j\omega}) = \begin{cases} -j, & 0 < \omega < \pi, \\ j, & -\pi < \omega < 0 \end{cases}$$

is referred to as a 90^o phase shifter and is used to generate what is referred to as an analytic signal $w[n]$ as shown in Figure (a). Specifically, the analytic signal w[n] is a complex valued signal for which

$$Re\{w[n]\} = x[n],$$

$$Im\{w[n]\} = y[n].$$

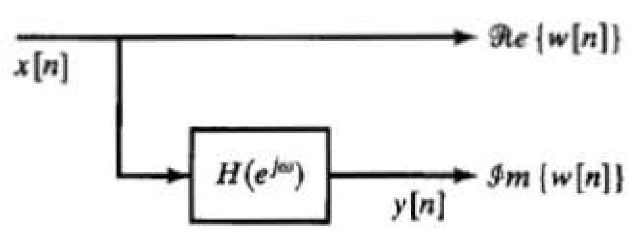


Figure (a)

If $X(e^{j\omega})$ is as shown in Figure (b), determine and sketch $W(e^{j\omega})$, the Fourier transform of the analytic signal $w[n] = x[n] + jy[n]$.

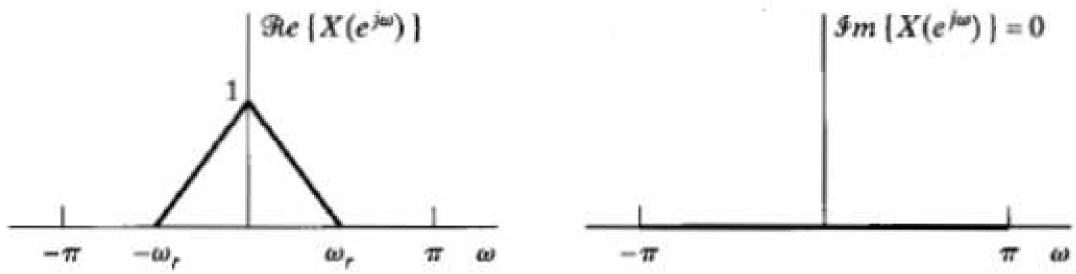


Figure (b)

5.Consider the following periodic signal:

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

(a) Determine and sketch its power density spectrum.
(b) Evaluate the power of the signal.

6. The frequency response of an ideal bandpass filter is given by

$$H(\omega) = \begin{cases} 0 & |\omega| \le \pi/8 \\ 1 & \pi/8 \le |\omega| \le 3\pi/8 \\ 0 & 3\pi/8 \le |\omega| \le \pi \end{cases}$$

(a) Determine its impulse response.
(b) Show that this impulse response can be expressed as the product of $\cos(\pi n/4)$ and the impulse response of a lowpass filter .

7. Determine the frequency response H(ω) of the following moving average filters.

(a) $y(n)= \frac{1}{2M+1}\sum_{k=-M}^{M} x(n-k)$

(b) $y(n)= \frac{1}{4M}x(n+M) + \frac{1}{2M}\sum_{k=-M+1}^{M-1} x(n-k) + \frac{1}{4M}x(n-M)$

MATLAB

1. Write a MATLAB function to compute the DTFT of a finite-duration sequence. The format of the function should be function [X] = dtft (x,n,w)

% Computes Discrete-time Fourier Transform

% X = DTFT values computed at w frequencies

% x = finite duration sequence over n

% n= sample position vector

%w= frequency location vector

2. For each of the linear time-invariant system described by the impulse response, determine the frequency response function H (e^{jw}) and plot the magnitude response |H (e^{jw}) | and the phase response ∠H (e^{jw}).

a. $h(n) = (0.9)^{|n|}$
b. h(n) = sinc (0.2n)[u(n+20) – u(n-20)], where sinc 0 = 1.
c. h(n) = sinc (0.2n)[u(n) – u(n-40)]
d. $h(n) = [(0.5)^n + (0.4)^n]u(n)$
e. $h(n) = (0.5)^{|n|} \cos(0.1\prod n)$

3.For a linear time-invariant system described by the difference equation

$$y(n) = \sum_{m=0}^{M} b_m x(n - m) - \sum_{l=1}^{N} a_t y(n - l)$$

the frequency response function is given by

$$H(e^{jw}) = \frac{\sum_{m=0}^{M} b_m e^{-jwm}}{1 + \sum_{l=1}^{N} a_l e^{-jwl}}$$

write a MATLAB function freqresp to implement the above relation. The format of this function should be

function [H] = freqresp(b,a,w)

% Frequency response function from difference equation

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%  [H] = freqresp(b,a,w)
%  H = Frequency response array evaluated at w frequencies
%  b = numerator coefficient array
%  a = denominator coefficient array (a(1) = 1)
%  w = frequency location array
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4. An ideal lowpass filter is described in the frequency domain by

$$H_d(e^{j\omega}) = 1.\, e^{-j\alpha\omega} \quad for\ |\omega| \leq \omega_c$$

$$and \quad H_d(e^{j\omega}) = 0 \qquad for\ \omega_c < |\omega| < \pi$$

where ω_c is called the cut-off frequency and α is called the phase delay.

a. Determine the ideal impulse response $h_d(n)$ using the IDTFT relation (3.2).

b. Determine and plot the truncated impulse response

$$h(n) = h_d(n) \quad for\ 0 \leq n < N\text{-}1$$

$$and \quad h(n) = 0, \qquad otherwise$$

for $N = 41$, $\alpha = 20$, and $\omega_c = 0.5\pi$.

c. Determine and plot the frequency response function $H(e^{j\omega})$ and compare it with the ideal lowpass filter response $H_d(e^{j\omega})$. Comment on your observations.