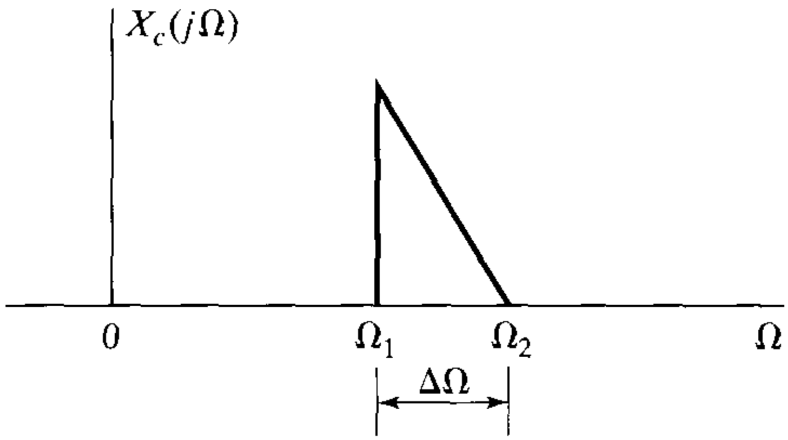
Assigment-1

1. A complex valued continuous-time signal $x_c(t)$ has the Fourier transform shown in Figure the below, where $(\Omega_2 - \Omega_1) = \Delta\Omega$. This signal is sampled to produce the sequence $x[n] = x_c(nT)$.



(a) Sketch the Fourier transform $X(e^{j\omega})$ of the sequence $x[n]$ for $T = \pi/\Omega_2$.

(b) What is the lowest sampling frequency that can be used without incurring any aliasing distortion, i.e., so that $x_c(t)$ can be recovered from $x[n]$?

(c) Draw the block diagram of a system that can be used to recover from $x_c(t)$ from $x[n]$ if the sampling rate is greater than or equal to the rate determined in Part (b). Assume that (complex) ideal filters are available.

2. Consider the sequence $x[n]$whose Fourier transform $X(e^{j\omega})$ is shown in Figure. Define

$$x_{s[n]} = \begin{cases} x[n], & n = Mk, \quad k = 0, \pm 1, \pm 2, ... \\ 0, & \text{otherwise}, \end{cases}$$

and

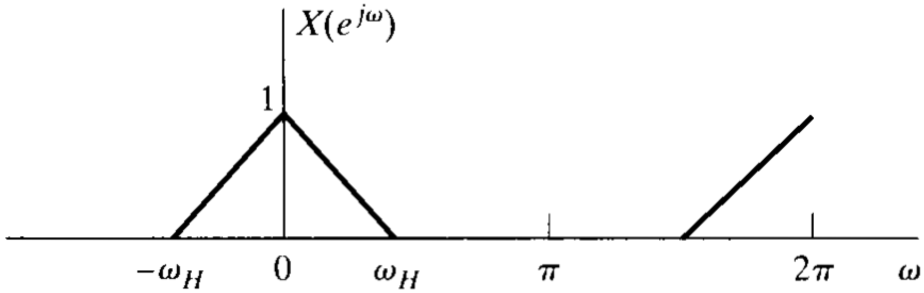
$$x_d[\mathrm{n}] = x_s[Mn] = x[Mn]$$

(a) Sketch $X_s(e^{j\omega})$ and $X_d(e^{j\omega})$ for each of the following cases:

(i) M=3,ω_H=$\pi/2$

(ii) M=3,ω_H=$\frac{\pi}{4}$

(b)What is the maximum value of ω_H that will avoid aliasing when M=3?



3. A bandlimited continuous- time signal $x_a(t)$ is sampled at a sampling frequency $F_s >= 2B$. Determine the energy E_d of the resulting discrete-time signal $x(n)$ as a function of the energy of the analog signal, E_a, and the sampling period $T = 1/F_s$.

4. Let $x_a(t)$ be a time-limited signal; that is, $x_a(t) = 0$ for $|t|>\tau$, with Fourier transform $X_a(F)$. The function $X_a(F)$ is sampled with sampling interval $\delta F = 1/T_s$.

(a) Show that the function

$$x_p(t) = \sum_{n=-\infty}^{\infty} x_a(t - nT_s)$$

can be expressed as a Fourier series with coefficients

$$c_k = \frac{1}{T_s} X_a(k\delta F)$$

(b) Show that $X_a(F)$ can be recovered from the samples $X_a(k\delta F)$, $-\infty<k<\infty$ if $T_s >= 2\tau$.

(c) Show that if $T_s < 2\tau$, there is "time-domain aliasing" that prevents exact reconstruction of $X_a(F)$.

(d) Show that if $T_s >= 2\tau$, perfect reconstruction of $X_a(F)$ from the samples $X(k\delta F)$ is possible using the interpolation formula

$$X_a(F) = \sum_{k=-\infty}^{\infty} X_a(k\delta F) \frac{\sin[(\pi/\delta F)(F - k\delta F)]}{(\pi/\delta F)(F - k\delta F)}$$

5. Figure 1 depicts a system for interpolating a signal by a factor of L, where

$$x_e[n] = \begin{cases} x[n/L], & n=0, \pm L, \pm 2L, \text{etc}\ldots, \\ 0, & \text{otherwise}, \end{cases}$$

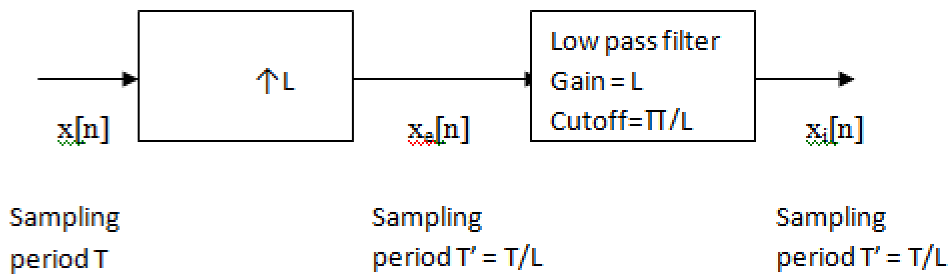


Fig 1 General system for sampling rate increase by *L*.

and the lowpass filter interpolates between the nonzero values of $x_e[n]$ to generate the upsampled or interpolated signal $x_i[n]$. When the lowpass filter is ideal, the interpolation is referred to as bandlimited interpolation. As indicated in section 4.6.2(Oppenheim, Schafer), simple interpolation procedures are adequate in many applications. Two simple procedures often used are zero-order-hold and linear interpolation. For zero-order-hold and linear interpolation, each value of x[n] is simply repeated *L* times; i.e.,

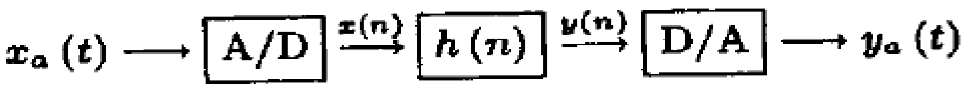
$$x_i[n] = \begin{cases} x_e[0], & n = 0, 1, \ldots, L-1, \\ x_e[L], & n = L, L+1, \ldots, 2L-1 \\ x_e[2L], & n = 2L, 2L+1, \ldots, \\ \cdot & \\ \cdot & \end{cases}$$

Linear interpolation is described in section 4.6.2(Oppenheim, Schafer).

a) Determine an appropriate choice for the impulse response of the low pass filter in Fig 1 to implement zero-order-hold and linear interpolation. Also, determine the corresponding frequency response.
b) Equation (4.92) (Oppenheim, Schafer) specifies the impulse response for linear interpolation. Determine the corresponding frequency response. (You may find it helpful to use the fact that $h_{lin}[n]$ is triangular and consequently corresponds to the convolution of two rectangular sequences.)
c) Sketch the magnitude of the filter frequency response for zero-order-hold and linear interpolation. Which is a better approximation to ideal bandlimited interpolation?

MATLAB Problems

1. We have the following analog filter, which is realized using a discrete filter



The sampling rate in the A/D and D/A is 100 sam/sec and the impulse response is
$h(n) = (0.5)^n u(n)$.

a) What is the digital frequency in $x(n)$ if $x_a(t) = 3\cos(20\pi t)$?
b) Find the steady state output $y_a(t)$ if $x_a(t) = 3\cos(20\pi t)$?
c) Find the steady state output $y_a(t)$ if $x_a(t) = 3\text{u}(\text{t})$?
d) Find two other analog signals $x_a(t)$, with different analog frequencies, that will give the same steady state output $y_a(t)$ when $x_a(t) = 3\cos(20\pi t)$ is applied.
e) To prevent aliasing, a prefilter would be required to process $x_a(t)$ before it passes to the A/D converter. What type of filter should be used, and what should be the largest cutoff frequency that would work for the given configuration?

2. Consider an analog signal $x_a(t) = \sin(20\pi t)$, $0 \le t \le 1$. It is sampled at T_s = 0.01, 0.05, 0.1 sec intervals to obtain $x(n)$

a) For each T_s plot $x(n)$.
b) Reconstruct the analog signal $y_a(t)$ from the samples of $x(n)$ using the sinc interpolation (use $\Delta t = 0.001$) and determine the frequency in $y_a(t)$ from your plot. (Ignore the end effects)
c) Reconstruct the analog signal $y_a(t)$ from the samples $x(n)$ using the cubic spline interpolation and determine the frequency in $y_a(t)$ from your plot. (Ignore the end effects)
d) Comment on your results.

3. Consider the analog signal $x_a(t) = \sin(20\pi t + \pi/4)$, $0 \le t \le 1$. It is sampled at T_s=0.05 sec intervals to obtain $x(n)$.

a) Plot $x_a(t)$ and superimpose $x(n)$ on it using the plot (n, x, ‘o’) function.
b) Reconstruct the analog signal $y_a(t)$ from the samples $x(n)$ using the sinc interpolation (use Δt = 0.001) and superimpose $x(n)$ on it.
c) Reconstruct the analog signal $y_a(t)$ from the samples $x(n)$ using the cubic spline interpolation and superimpose $x(n)$ on it.

You should observe that the resultant reconstruction in each case has the correct frequency but different amplitude. Explain this observation. Comment on the role of phase of $x_a(t)$ on sampling and reconstruction of signals